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The fascination of fluid mechanics

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Two topics are discussed in order to illustrate the author's own enjoyment of fluid mechanics. The first and longer discourse is about splashes. It makes no attempt at completeness but includes a little new research. The second part deals briefly with many variations on the theme of flow in pipes.

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1. Prologue

To be fascinated by something implies that one is attracted to it, possibly in spite of some less pleasant features. For those of us who work in the domain of fluid dynamics the less pleasant features may be tedious time-consuming analysis or experiments, or the difficulties one meets in trying to understand or predict fluid flows. This is enough of the less pleasant side; difficult problems attract people by the challenge of their existence. Even tedious work can be a 'restful' interlude between more demanding tasks. This essay gives an indication, rather than an explicit account, of the ways which fluid mechanics fascinates its author.

What a person enjoys and is interested in is very much a matter of individual taste. This shows up very clearly in their choice of music (one aspect of fluid mechanics!).

Some people like to 'feel' the sound, one choosing the strongly amplified popular music that has become available with modern technology, another choosing 'old technology' and revelling in the thunderous interplay of notes produced by a large organ and their echoes from the massive structure of church or cathedral. Others prefer to listen to the varied talents of the human voice, and there are only a few who do not gain enjoyment from some sort of music.

Our perception of the world is dominated by our vision. The commonest fluids, air and water, are transparent, so we most readily see the interface between them. The air–water interfaces of clouds, and rain, of streams, rivers, lakes and the seas are vital elements in most of the natural scenes we enjoy. The first topic to be dealt with in this essay is that of splashes, perhaps the most eye catching of interfacial phenomena but one which has received little attention. Parts of this section have 'grown' in the preparation of this work. Indeed, one portion has grown to the point where it had to be cut out and written up separately. This is a pity, in a way, since it refers to both the papers contributed to the first volume of this journal by its editor. There are a number of questions raised and problems left untouched in this section. I hope some readers may care to study them further and let me know how they fare.

The second topic appears more mundane: the flow of fluid in pipes. However, there is a wealth of variety and interest in such flows which may not be immediately apparent and is briefly sketched.

2. Splashes

One of the simplest fascinating aspects of water is the ease with which it can be splashed. Babies a few weeks old can delight in splashing in their baths. Once they are mobile they can think of little better than playing with liquids, preferably muddy, pouring them and splashing them. Older children and exuberant adults also delight in splashing each other in play, or throwing stones to make splashes in otherwise tranquil water.

As well as the active pleasures of water splashing, there is appreciable passive appreciation of it. It may come indirectly from the pleasure of watching young children at play but can also come directly. Among the most awesome and spectacular displays for spectators on earth are large waterfalls. The fall of water is only part of the spectacle. Usually the water is splashing long before it hits the bottom and this contributes to the noise and ground vibration which can make a great fall truly fascinating. At coastal locations also much pleasure can be gained by watching waves with their splashing white breakers as they come up a beach and the splashes made as they hit cliffs or coastal structures.

To the more professional eye a splash may seem to be a very simple example of fluid mechanics. The water is projected into the air and soon breaks into drops which follow a trajectory which is more or less determined by the methods of particle mechanics. To some extent this view does survive a closer investigation, nevertheless there are aspects of, and diversions from, the problem which add interest to it.

2.1. *Breaking waves*

My own technical interest in splashes arises from attempting to understand the fluid mechanics of water waves. It is now a few years since we have been able to describe in detail the water motion in a wave as it is about to break (Longuet-Higgins & Cokelet, 1976). However we do not understand *why* a jet of water is projected from the wave crest in such a characteristic manner. For an account of our present understanding, which is more descriptive than predictive, see Peregrine, Cokelet & McIver (1980). The splash first arises when the falling jet of water from the crest of the wave hits the undisturbed water in front of it. Sometimes the splash rises to a greater height than the original wave.

To study such a splash it is natural to commence by making as much simplification as is possible whilst still retaining the dominant physical influences. If the scale of the breaking jet is large enough it seems quite reasonable to neglect viscosity and surface tension. The effect of the air can also be neglected, although it is easy to see that the presence of air is often an important feature.

During and shortly after the first impact of a falling jet on undisturbed water the effect of gravity is likely to be small. Gravity certainly affects the trajectory of the falling and the splashed water, but need only be taken into account in determining conditions before impact, and in the subsequent behaviour once conditions of projection of the splash are known.

Even with simplifications like these the problem is still a difficult, two-dimensional unsteady free-surface problem. We still know insufficient about the falling jet to make a highly detailed analysis worth while. Further simplification is possible.

2.2. *Mathematical model of a splash from a thin layer of water*

A simple, not entirely unrealistic, way of describing the jet is as a 'moving waterfall' which is 'switched on' at some initial instant. Suppose it moves with constant velocity V_1 relative and parallel to the water it hits and has constant properties once it hits the water. By further assuming the jet is thin it is readily characterized by its thickness h_1 and its water velocity parallel to its surface, u_1, immediately above the impact point. This is a one-dimensional representation which could be improved by letting V_1, h_1 and u_1 be functions of time if more were known about the jet.

The impact and splash is still two-dimensional, but by taking a special case this too can be made one-dimensional. That is if the depth of undisturbed water h_0 is small compared with the dimensions of the breaking wave forming the falling jet. That is is $u_1 \gg (gh_0)^{\frac{1}{2}}$. This is often realistic for waves breaking on steep beaches. The neglect of gravity in the impact process means that the slope of the beach or any uniform velocity in the water does not enter this aspect of the problem.

A one-dimensional model describes the water just by its depth or thickness and its velocity. Thus in figure 1 a possible initial development of the splash is shown. The originally undisturbed water is taken to have depth d_0 and to specify a reference frame with zero velocity. The velocities u_1, u_2 and u_3 are velocities of the projected or falling fluid relative to its surface while V_1, V_2 and V_3 are the velocities of the points of impact or projection. Thus at impact the velocity of the approaching water is $(V_1 - u_1 \cos A_1,\ -u_1 \sin A_1)$ in this reference frame. The velocities v_1 and v_2, depths d_1 and d_2 and angles A_1, A_2, A_3 are all defined in figure 1.

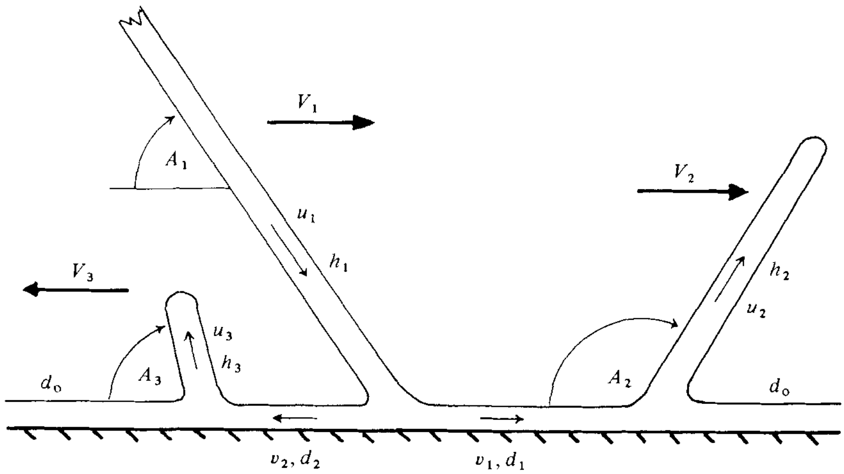

FIGURE 1. One-dimensional model of the splash of a 'moving waterfall' onto a thin layer of water, defining depths, d_i, thicknesses, h_i, velocities, u_i, V_i, and angles A_i.

This model is complete once relations are found between the three streams, at the impact point and the projection points. These relations are readily found once it is noted that at each of these points the configuration is similar to that of the symmetrical impact of two plane jets (e.g. see Milne-Thomson 1960, §11.42, interestingly it is noted in §11.40 that only the symmetric solution is determinate). In the reference frame moving with the impact or projection point the flow in that neighbourhood is steady. Bernoulli's theorem holds along stream lines and with our neglect of gravity reduces to constant velocity on surface streamlines. That is

$$u_1 = v_1 - V_1 = v_2 + V_1, \tag{1}$$

$$u_2 = v_1 - V_2 = V_2, \tag{2}$$

and

$$u_3 = v_2 - V_3 = V_3. \tag{3}$$

The flow into and away from these points must conserve mass and this reduces to

$$h_1 = d_1 + d_2, \tag{4}$$

$$h_2 = d_1 + d_0 \tag{5}$$

and

$$h_3 = d_2 + d_0. \tag{6}$$

Similarly the rate of flow of horizontal momentum is conserved, which simplifies to

$$h_1 \cos A_1 = d_1 - d_2, \tag{7}$$

$$-h_2 \cos A_2 = d_1 - d_0 \tag{8}$$

and

$$-h_3 \cos A_3 = d_0 - d_2. \tag{9}$$

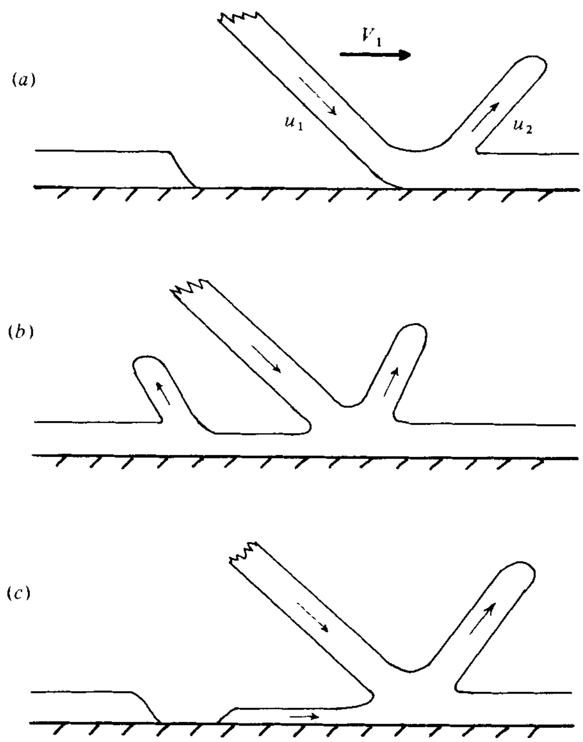

FIGURE 2. Alternative splash configurations.

These nine simple algebraic relations mean that all quantities can be found in terms of the 'initial' values d_0, h_1, u_1, V_1 and A_1. The velocities are

$$v_1 = u_1 + V_1, \quad v_2 = u_1 - V_1, \tag{10), (11}$$

$$V_2 = u_2 = \tfrac{1}{2}(u_1 + V_1), \tag{12}$$

$$V_3 = u_3 = \tfrac{1}{2}(u_1 - V_1). \tag{13}$$

The linear relations between the velocities and our assumption of constancy means that the distances between impact and projection points simply increase linearly with time after the initial impact. This indicates that a corresponding fully two dimensional problem might be solved using variables $(x/t,\ y/t)$.

The model configuration proposed in figure 1 only makes sense if all the velocities are positive. This requires $u_1 > V_1$. If $u_1 < V_1$ no water is sent 'backwards' from the point of impact. Also, $V_2 - V_1 = \tfrac{1}{2}(u_1 - V_1)$ is also negative and the projection point cannot travel ahead of the impact point. A rough estimation indicates that $u_1 > V_1$ and $u_1 < V_1$ are both possible for breaking waves. Perhaps the configuration sketched in figure 2(*a*) may be relevant in the case $u_1 < V_1$.

Surprisingly, the configuration of figure 2(*a*) is also determinate in a frame of reference where the motion is steady. But, from Bernoulli's theorem

$$u_1 = u_2 = V_2,$$

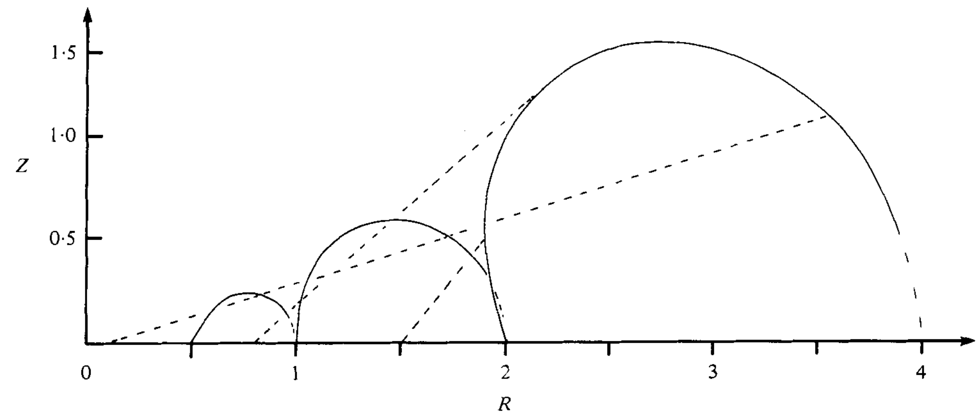

FIGURE 3. Splash from a falling cylindrical jet with negligible gravity; a section through a radial plane, at three times. The broken lines show particle trajectories.

so that it can only apply for the special case $u_1 = V_1$. The indeterminate configurations of figures 2(*b*) and (*c*) suffer the same defect.

One, as yet unmentioned, approximation ought to be brought in here. All the above analysis has assumed irrotational flow for the various parts of the splash. However the flow is not generally irrotational. When two bodies of water meet, a vortex sheet forms at the dividing surface. Its strength depends on the relative tangential velocity at each point just before contact. Thus the model under discussion strictly only applies when $u_1 \cos A_1 = V_1$, otherwise some account should be taken of the vortex sheet.

One case where the model should hold is for water which simply drops vertically onto a shallow layer of water. The simplest experiment of this sort is to turn a tap on when there is a thin layer of water beneath it. For such an axisymmetric case the splash arises from a circular line of increasing radius. The spreading horizontal sheet of water pushing up the splash has an almost constant velocity if viscous effects are ignored but a decreasing thickness, d_1 (corresponding to d_1 in figure 1). Since from equations (5) and (8)

$$\cos A = \frac{d_0 - d_1}{d_0 + d_1}, \tag{14}$$

the angle of projection of the splash varies with position. Here it is assumed that conditions on the circular projection line are adequately described by supposing it to be locally plane.

If the constant volume flux of the axisymmetric falling jet is Q, its velocity at impact with a horizontal plane is u_1 and r, z, t are radial and vertical co-ordinates and time then the problem can be made dimensionless by putting

$$(r, z) = (Q/2\pi u_1 d_0)(R, Z) \tag{15}$$

and

$$t = QT/\pi u_1{}^2 d_0. \tag{16}$$

Then we have

$$d_1 = d_0/R. \tag{17}$$

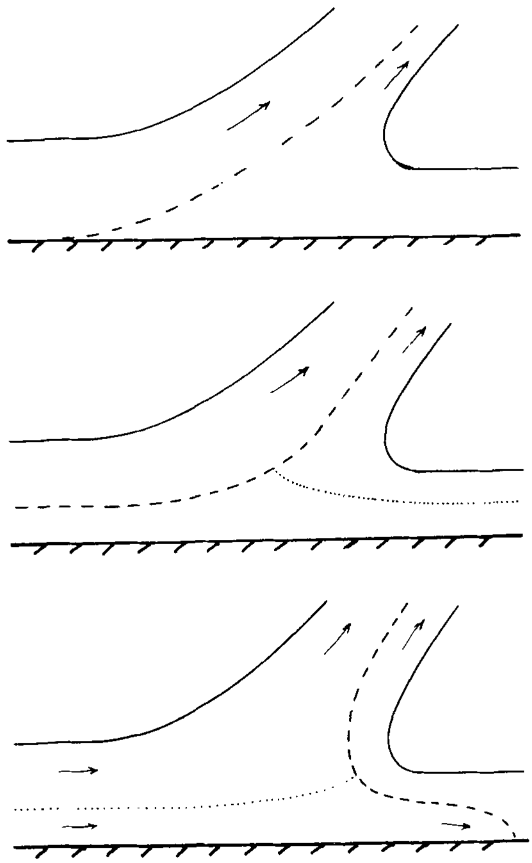

FIGURE 4. Possible streamlines at splash separation with a vortex sheet, shown as a broken line. Other dividing streamlines are shown with lines of dots.

Assuming a simple ballistic trajectory for liquid particles once projected upward leads to an equation:

$$(R, Z) = \left(T_1 + \frac{2(T-T_1)}{1+T_1}, \frac{2(T-T_1)}{1+T_1} T_1^{\frac{1}{2}} - \tfrac{1}{2}G(T-T_1)^2\right), \tag{18}$$

where $0 < T_1 < T$, for the shape of the projected splash. It depends on a single parameter $G = 2gQ/(\pi u_1^3 d_0)$ which gives a measure of the significance of gravity. Figure 3 shows the shape of the splash (18), for $G = 0$, at $T = 0{\cdot}5$, $1{\cdot}0$ and $2{\cdot}0$. The jet falls down the Z axis. The broken lines indicate the trajectories of liquid particles. There would be little difference in the shape for small values of G. It is possible to calculate the thickness of the splash but this has not been done.

2.3. *Viscous effects in the unsteady separation of a thin splashing layer*

When a vortex sheet is formed it is more difficult to find a steadily moving flow pattern to describe the projection of a splash. There can only be a stagnation point on one side of the vortex sheet. Some possible streamlines are sketched in figure 4, which includes sketches drawn with the 'hindsight' of having considered viscous

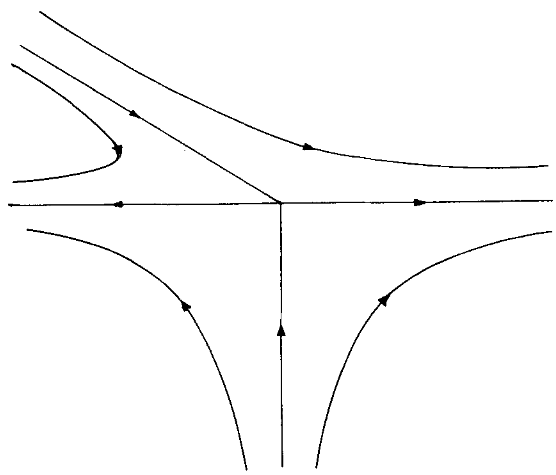

FIGURE 5. Inviscid flow near a stagnation point with vorticity in $y > 0$.

flows; but, after all, viscosity or instabilities cannot be neglected in a real vortex sheet.

As an initial assumption let us suppose the flow is such that the effects of viscosity are only cumulative. That is, viscosity causes the development of a velocity profile in outward spreading water but the splash separation is almost inviscid since its time scale is shorter. The flow is separating from a moving point on the plane so that it is an example of 'unsteady separation'. However, if we take the case where the simple model of the previous section is appropriate then it can reasonably be considered stationary in the frame of reference moving with the separation point.

For separation to occur a streamline must split. This can only happen at a stagnation point. It just does not seem possible for a smooth bifurcation of a streamline to occur in normal circumstances. For example, see the discussion of low-Reynolds-number eddies by Jeffrey & Sherwood (1980). Yet examples of bifurcating streamlines do occur in inviscid flow solutions; for example, Hopkinson (1898) gives some free-streamline examples and Pierrehumbert (1980) finds an example at the boundary between irrotational and rotational flow for a translating vortex pair.

The liquid which is at rest relative to the plane has zero vorticity while the spreading liquid has some viscosity-induced vorticity. Consideration of the directions from which fluid approaches the dividing streamline in the steady frame indicates that this streamline also divides the two regions of different vorticity. An inviscid solution for flow near a stagnation point dividing two regions of vorticity is readily found. The stream function

$$\psi = \begin{cases} Axy + \tfrac{1}{2}\Omega y & \text{in} \quad y > 0 \\ Axy & \text{in} \quad y < 0 \end{cases} \tag{19}$$

gives irrotational flow in $y < 0$ and flow with vorticity $-\Omega$ in $y > 0$. Across the dividing streamline $y = 0$, pressure and velocity are continuous. The flow pattern is sketched in figure 5. The streamline which approaches the stagnation point in the upper half-plane is $y = -2Ax/\Omega$.

After a little consideration this solution leads to the sketch of a plausible streamline pattern given in figure 6 for the flow in the 'steady' reference frame. This clearly

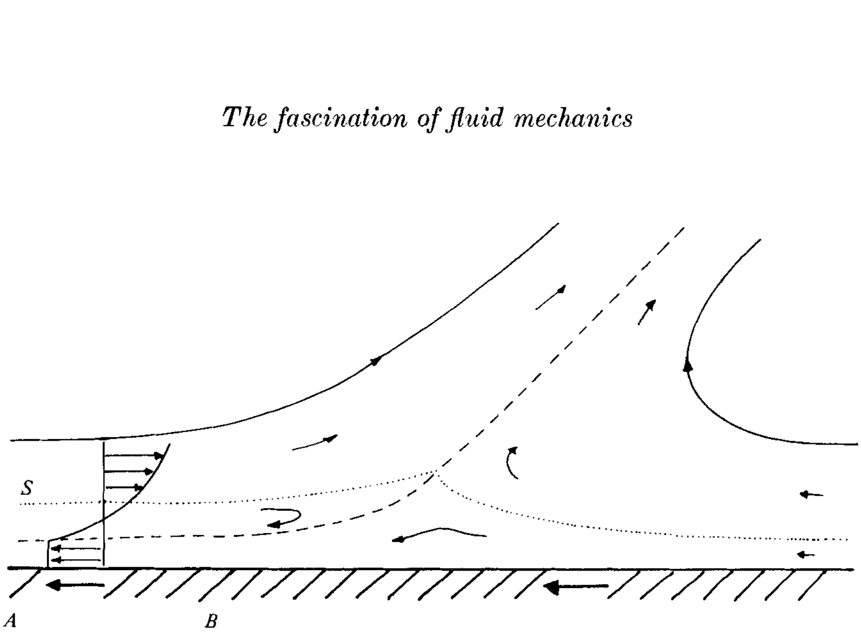

FIGURE 6. Sketch of the inviscid separating flow field in the frame of reference moving with the splash, when a velocity profile has developed in the incoming flow.

indicates some of the problems of unsteady separation. In particular, the separation streamline S is not readily identified in the incoming flow, because some of the incoming fluid returns from the vicinity of the stagnation point. Thus, the streamline which 'enters' with zero relative horizontal velocity is not the dividing streamline. Some of the fluid which was at rest is being overrun by the splashing liquid. Indeed, if the Reynolds number of this portion of the flow is really large the velocity profiles along AB could look like that sketched in figure 6.

If the viscous effects are entirely confined to a thin boundary layer, these effects are less prominent but must still exist. At a relative stagnation point in the boundary layer the vorticity Ω must be very large compared with the rate of strain, A, so that the streamlines through that point enclose a very small angle. The flow almost certainly looks like the examples of boundary-layer 'separation' off a rotating cylinder photographed by Koromilas & Telionis (1980, figures 10 and 11), and Morton (1980, private communication).

Looking at this type of flow field it is difficult to see the relevance of the criteria for unsteady separation suggested by Moore, Rott and Sears who add a condition $\partial u/\partial y = 0$ at the relative stagnation point (see for example the review by Williams 1977, as well as Koromilas & Telionis 1980). The case where the simple model of §2.2 is appropriate is one where a solution for the outer potential flow exists (Milne-Thomson 1960, §11.34) and is probably appropriate if the separating streamline emerges from the boundary layer with only a narrow wake-like region of vorticity. Hence it may prove valuable for a deeper study of unsteady separation.

In general there will be a shear across the region of the dividing streamline. If viscous effects are strong this soon diffuses across the whole jet. If they are weak the resulting turbulent mixing layer will rapidly have the same effect and probably begin to break up the projected sheet of liquid.

2.4. *An exact solution of the Navier–Stokes equations*

The inviscid stagnation point flow described by equation (19) can easily be extended to include viscosity. The flow toward the separating streamline acts against the diffusion of vorticity. A trial velocity field of

$$(u, v) = (Ax + U(y), -Ay) \tag{20}$$

inserted in the Navier–Stokes equations, leads to an exact solution with vorticity, $U'(y)$, having a smooth error function profile between any two constant values as $y \to \pm\infty$. That is,

$$U'(y) = B\,\mathrm{erfc}\,[y(A/2\nu)] + C, \tag{21}$$

where ν is the kinematic viscosity and B and C are constants determined by the vorticity at $y = \pm\infty$.

It is a pleasant surprise to find a simple exact solution of the Navier–Stokes equations. Further research leads, via the comprehensive article by Berker (1963), to Jeffery (1915) as the first person to find this solution. His opinion was, 'Solutions I, II [this solution], and IV lead to some interesting sets of streamlines. They cannot, however, be realized physically, and they seem to be of little importance'.

However, the solution may be of some value in the context of unsteady separation and there is another flow where it may also be relevant. This is the steady laminar high-Reynolds-number flow about a cylindrical bluff body. Smith (1979) presents a detailed theoretical model of such a flow with particular emphasis on the separation region and the size of the eddies which form behind the cylinder. One admitted weak point in this account is that the rear stagnation point and internal flow of the eddies are not fully elucidated.

At the rear of the eddies flow is converging towards the dividing streamline from both sides with vorticity of opposite signs. Thus the flow in the neighbourhood of the stagnation point should be described by the solution (21) with appropriate constants.

Fornberg (1980) investigates the flow about a circular cylinder by accurate numerical integration of the Navier–Stokes equations. The results of the computations are in striking disagreement with Smith's (1979) hypothesis about the eddies at the highest Reynolds numbers calculated. A note discussing this flow has been prepared (Peregrine 1981).

2.5. *The weakest splashes*

After the digression of the previous section we return to the theme of splashes and briefly consider the very weak splashes formed by a single drop of liquid dropped from a height much less than a metre. There is no discernible splash if a drop is placed on the surface of water, yet even then there is appreciable water motion. If there are no strong surface active effects the combined effects of surface tension and gravity act to create a vortex ring. This is illustrated in Batchelor (1967), plate 21, for a drop starting 10 mm above the surface. The vorticity may be due to the relative velocities of surfaces as they meet, but could also be due to the effects of surface tension immediately after contact, when there must be some near-singular motion. (See also § 3.2 of this essay.)

I shall refrain from a digression into the many, varied and unexpected properties of vortex rings. Some of these are quite well known. However, the further development of the vortex ring one gets from such drops is worth noting. Clearly, to visualise the flow, dye or some other substance must be used. Small concentrations of such a marker substance are seen to 'tighten up' at a few points around the ring and form further small rings which descend a few centimetres before they too are subject to the same fragmentation. A whole cascade of such events can form in suitable circumstances. It is not clear whether a density difference in the fluids is necessary to develop this beautiful motion, but it does seem likely. It is demonstrated in the opening sequence of the film 'Flow instabilities' by Møllo-Christensen (1969). In the accompanying booklet he recommends a cream (50 %)–milk (50 %) mixture for drops falling into cold water. A drop of ink is often satisfactory.

If the same experiment is performed with a detergent solution as the receiving fluid, the drop and the fluid frequently do not coalesce. The drop may bounce and run freely over the surface. A steadily dripping tap falling into a washing-up bowl can produce quite a regular supply of these free drops. Although the liquids do not make contact through the surface film of detergent the bulk liquid surface supports the weight of the drop and is thus depressed beneath it. Hence drops attract each other and merge if they do not contain detergent. On the other hand bubbles sitting on the surface repel drops since surface tension raises the surface in their vicinity. It is an intriguing dynamical system to see, with drops and bubbles repelling each other but being mutually attracted. The drops have substantial inertia compared with the bubbles. (A simple approximation for the attractive force between bubbles on a surface is made by Nicolson 1948.) Unfortunately the lifetime of drops on a surface is not often more than about ten seconds. If you are lucky you may see a bubble riding on top of a drop.

The same property of drops bouncing off a 'liquid' surface is even more readily demonstrated with drops blown onto a bubble formed from the same liquid. Drops can also go through the bubble without upsetting it. The same thing also happens with bulk liquid and can best be observed if it is in a transparent container. Drops which enter a liquid without coalescing with it form 'anti-bubbles'; that is, a thin spherical film of air separating two regions of water. The most notable properties of anti-bubbles compared with bubbles are their appreciable inertia. Their life time is similar to, or perhaps longer than, that of drops on a surface, but not as long as that of bubbles. Baird (1960) describes theory and experiment for anti-bubble life-times. He notes they are more stable in fresh tap water which is supersaturated with air which helps maintain the air film by coming out of solution.

2.6. *The projected water*

First consider what happens to a relatively compact mass of water projected into the air. For moderate velocities and small massse, about 1 gm, the air itself is unimportant and the liquid is in free fall. Thus if the water has little motion relative to its centre of mass the dominant force on it is surface tension and it is likely to oscillate about a spherical shape.

On the other hand larger masses of water, of 100 gms or more are unlikely to be fully constrained by surface tension forces. The inertia of the fluid due to pre-existing internal motion, which might be due to turbulence or to details of its projection, will dominate. There is no significant restraining force acting, so any surface portion of the mass which initially has a velocity directed away from the main bulk of water con-

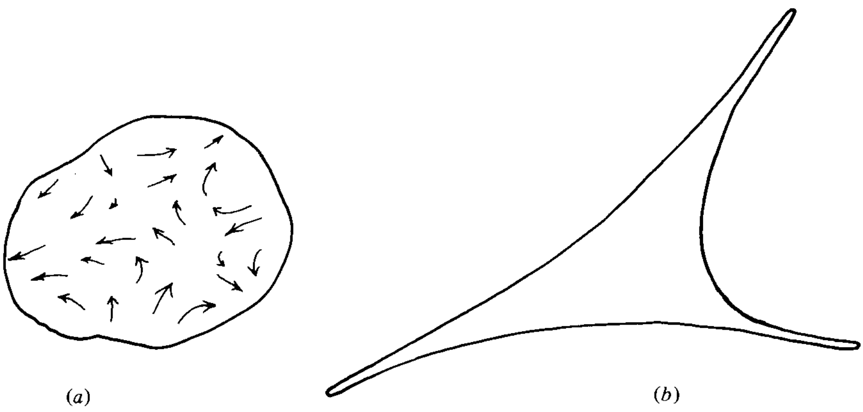

FIGURE 7. A projected mass of water (*a*) initially, (*b*) after some time.

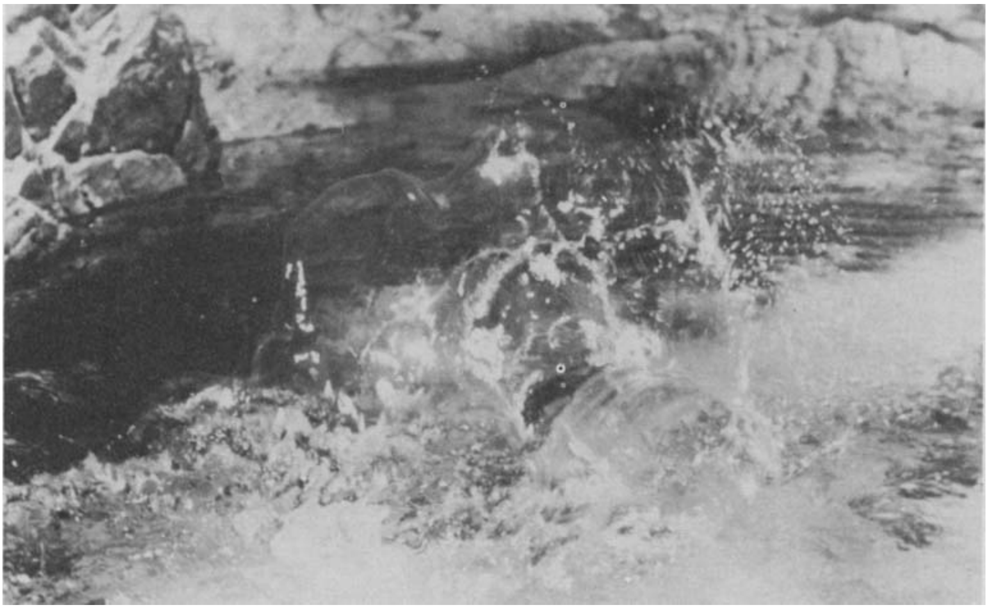

FIGURE 8. A 'water bubble'.

tinues to move away and the water spreads out into sheets and filaments of water. See figure 7 for a before and after sketch of such motion in two dimensions. It is probably the pre-existing turbulence in a river which cause seven thick water-falls to break up into spray quite rapidly.

This spreading of a fluid mass can be readily demonstrated with a small spade, or large 'seaside' spade, which has no raised edges. It should be held with the blade horizontal about 2–3 cm beneath the surface of still water. Then with a short sharp motion project the mass of water above the blade into the air, in order that it should reach a height of about one metre. The air should be still, or else the projection should include a component of velocity in the downwind direction.

During the initial acceleration, while the water is on the spade, it begins to flow towards the edges of the spade's blade. Once it is projected and is in free fall that motion continues and by the time the water reaches the top of its trajectory it has spread out into a large sheet several times its original horizontal extent. This large sheet then descends, trapping a large volume of air, see figure 8. My children consider this to be a fine game at the seaside and have christened the resulting sheets 'water bubbles'. It takes little practice to produce water bubbles almost a metre across. In still air they often maintain their integrity until they are thin enough to show interference colours. It is also often possible to see the way in which such a sheet of water breaks up into drops.

This occurs mainly at the edge. A sheet of water appears to be quite stable unless it is very thin, or travelling so fast that there is strong interaction with the air. A full account of the disintegration of thin fast-moving sheets is given by Dombrowski & Fraser (1954).

At the free edge of a sheet of water, surface tension acts to minimize the surface area. This leads to the formation of a cylindrical bulge moving into the sheet. This can be described by analysis similar to that of Fraser *et al.* (1962) for the growth of a hole. Short waves can often be seen ahead of it; these cause a 'wave resistance' to its motion but are unlikely to be more than minor features. An isolated cylindrical column breaks up into drops due to a classical instability described by Rayleigh (1945, §357 onwards). It appears that this cylinder on the edge of a sheet of liquid suffers from the same instability and breaks into drops.

The best known examples of this instability are the 'crowns' which form in the initial splash from a drop. These were first photographed in the pioneering work by Worthington (1897, 1908). They show up best when the drop falls on a very thin film of liquid.

As well as a primary impact splash, most splashes caused by a single drop or other mass have a secondary phase as water returns to the point of impact and rises in a symmetrical column. For a single drop this usually breaks up to project another drop, of almost the identical fluid, back up into the air. If the original drop falls into water near the edge of a full water container the resulting secondary splash is asymmetrical and can consistently throw the second drop over the side of the container. This is very probably an effect of the same kind as the 'microjet' which is directed towards a nearby wall as cavitation bubbles collapse. It appears that cavitation damage is not caused by the jet but by shock waves (see Fujikawa & Akamatsu 1980).

For most splashes the main effect of the air is its influence on the trajectories of sheets and drops of water due to the drag caused by relative motion. At the same time there is an equal and opposite drag on the air changing its motion. This mutual interaction is most apparent near the base of waterfalls where the falling water induces a substantial air current which spreads out horizontally at the base carrying drops of water outward. But this is not all, one of the more spectacular air–water interactions occurs when the relative velocity between a drop and the air increases beyond the range normal in everyday splashes. Large water drops explode!

2.7. *Exploding water*

Exploding water is readily demonstrated from any building three or more floors high. Just project a large drop out of the window. I find that the easiest way to do this is to take a plastic coffee cup with a centimetre or so depth of water at the bottom. With a little practice it is possible to throw a drop of about 1–2 cm diameter slightly upward.

One can see it oscillate in shape at the top of its trajectory and as it starts to descend. Then, after falling about 5 or 6 metres, it 'explodes' into a shower of smaller drops. Drops which are just too small to explode simply bifurcate.

What happens is that the air pressure deforms the drop until it is like an open bubble which becomes so thin that it bursts. The bubble is open at its bottom end, see the article by Lane & Green (1956). The resulting smaller drops spread out with quite an appreciable transverse velocity. It is not clear to me how this transverse velocity, which gives the appearance of an explosion, arises.

It is this type of air–water interaction which ensures that rain drops have a maximum size and that high waterfalls almost entirely break up into small drops. Other examples of natural splashes rarely reach such relative velocities, however photographs of very large breaking waves (perhaps > 6 m high) show a character to the spray which is very similar to the spray caused by real explosives. It may be that such breakers have passed some critical relative air velocity threshold.

2.8. *Large splashes*

Consideration of size leads to the largest splashes I am aware of. The largest possible are of a cosmic scale which we are unlikely to see. Impacts between celestial bodies do occur. The more recent of the large impact craters on the moon show traces of the splashes that occurred. In particular the bright rays from the crater Tycho stretch 2000 km across the face of the moon. It is also suggested with good evidence (Alvarez *et al.* 1980; Ganapathy 1980; Smit & Hertogen 1980) that an impact on earth 65 million years ago of a 10 km diameter meteorite caused the demise of the dinosaurs and many other species. On top of severe initial disturbance of the atmosphere and ocean, the debris from the impact was suspended in the stratosphere, severely attenuating sunlight so that the whole ecosystem of Earth was knocked off balance.

Although the splash from such an impact would be spectacular it is restricted by the atmosphere to a very short range. The cause of debris rising high into the atmosphere is the large amount of kinetic energy which is converted into heat. Once geologists identify the site of such an event (e.g. could it be at the antipodes of the Tertiary volcanic rocks of the Inner Hebrides?) then it is possible to compute the way in which waves would have propagated around the world's oceans.

No such splash has occurred in historic times on Earth. The highest well documented splash is described by Miller (1960). A landslide, caused by an earthquake, sent a large quantity of rock down into Lituya Bay, a fiord on the Alaskan coast on 9 July 1958. The resulting splash of water rose 525 metres above sea level on the opposite side of the fiord. This was not observed directly but was readily visible afterwards because the splash completely removed trees and soil from the area it covered. The resulting wave which travelled down the fiord was seen and 'ridden' by people in 3 boats, two of which foundered. Eye witness accounts are in Miller's (1960) description as well as photographs of the area affected.

2.9. *An atmospheric effect of splashes*

Finally there is one feature of splashes which we do not see, or hear. There is an extra 'freshness' to the atmosphere near waterfalls and the sea, and even near water sprinklers. It is almost certainly due to electric fields that arise from charge separation as water drops collide or split. Presumably we sense it by our organs of smell, and electrically produced ozone is one suggestion; but maybe we are sensitive in other ways to electric fields.

3. Pipe flows

3.1. *Laminar and turbulent flow*

The flow of viscous fluid through circular cylindrical pipes is one of the first practical problems encountered in the study of fluid dynamics. The Hagen–Poiseuille solution for laminar flow involves only simple mathematics, is relatively easily understood, is readily verified experimentally and has many direct applications.

For short pipes, inviscid flow can be a good approximation. Mass conservation and Bernoulli's theorem are then sufficient to account for the flow behaviour in pipes of slowly varying cross-section. The 'simple' property that pressure is less where the fluid velocity is greater often seems to be a paradox to people with no experience of fluid dynamics, and there are some delightful demonstrations which can help enliven introductory courses. For example, a light plastic ball can be held in an inverted funnel by blowing *down* it.

Further study of fluid dynamics leads to an introduction to turbulent flow. If a student is fortunate he or she may see a reproduction of Reynolds' (1883) famous experiment and see the startling change in flow behaviour. Failing that the film 'Turbulence' by Stewart (1969) gives a clear demonstration. Further study and experience brings familiarity with this and other transitions to turbulence, but nonetheless it is a remarkable transformation to see. A measure of its interest and importance is that even 100 years after the experiments, which were performed in 1880, work continues in order to improve our understanding of this flow. This is hardly surprising when one considers that since the beginning of this century turbulent flow has been considered to be one of the most difficult of physical problems.

We are still a long way from understanding turbulent flows. To some extent we qualitatively understand what is happening but it is not possible to predict turbulent flows except in those well defined cases where numerous experiments have led to reliable empirical rules. Happily many practical cases, such as flow through pipes, do lie within this field, at least for Newtonian fluids.

If someone has become fully familiar with turbulent pipe flows and the large increase in drag associated with turbulence, it can come as a considerable surprise to find that a tiny amount of an additive can reduce the drag substantially. For example, 20 parts per million of polyox, a long chain polymer, can reduce drag by 50 %. Berman (1978) reviews the subject.

Concentrated polymer solutions, or molten polymers, have many even more striking phenomena associated with their flow through pipes. Many involve their elastic behaviour, this includes a 'recoil' effect when a driving pressure gradient is stopped and the 'die-swell' effect where fluid swells considerably after flowing out of a tube. For these and other flows the book by Bird, Armstrong & Hassanger (1977), chapter 3, is a good source.

3.2. *Shock waves and vortex rings*

For compressible fluids there is also a transition to turbulent flow, but with the added complication of variable density and temperature. Perhaps the dominant aspect here is that exit velocities from a uniform pipe are limited by the velocity of sound in the gas. Many text books on compressible flow ignore this particular problem, but Shapiro (1953) gives a full account of it.

There are other aspects which can dominate gas flow. Turbulence in the flow, and particularly fluctuations of input conditions, lead to noise. In large industrial installations sound levels can reach 200 dB. That means the sound intensity is 10^{20} times the 0 dB standard, which corresponds, for certain loudness scales to a root mean square pressure fluctuation level of $2 \times 10^{-5} N\,m^{-2}$. It takes little calculation to show that 200 dB corresponds to sound waves with pressure variations greater than one atmosphere. Clearly such waves are not linear and rapidly steepen to form shock waves. If the existence of such shock waves is not allowed for then pipes can fail due to metal fatigue from the shock-induced stresses.

Unsteady flows are far more likely to develop shock waves. Even the humble domestic water supply can be troubled by the phenomenon which is then known as water hammer. This effect is usually deleterious, but is put to good use in the device known as a hydraulic ram. This consists of a length of pipe; at the low pressure end water is allowed to escape through a valve which automatically stops the flow as it approaches a preset value. The high pressure that results at this lower end, due to the rapid change of momentum of the water in the pipe when it is stopped is then used to pump some water to a very much higher level or pressure than was available in the original pipe. Water in the pipe then starts to flow again and the sequence is repeated with a frequency of order 1 Hz. The device can pump a useful amount of water from any stream or river by using the low head available in the streamwise direction. It does not seem to be making any comeback despite the present awareness of energy conservation. Presumably this is due to the capital costs involved, but may be due to ignorance or lack of supply. The hydraulic ram is not mentioned in current textbooks but older works, such as Gibson (1930), give full details.

Shock waves are induced deliberately in flows. The theory of the production of supersonic flows through a Laval nozzle is standard undergraduate material. The supersonic flow region is terminated by one or more shock waves. The achievement of high supersonic velocities is often hindered by the drop in temperature associated with the flow expansion since it can lead to condensation of the gas. For this and other reasons, shock waves are commonly studied in pipes of constant cross-section known as shock tubes. One of the most intriguing of experimental results to come from a shock tube is reported by Dettleff *et al.* (1979).

The compression behind a shock wave can be sufficient to cause liquefaction. This contrasts with condensation due to expansion and for most gases it does not occur because of the temperature increase. It is pointed out that Landau & Lifshitz (1959) state that it cannot occur. However for certain gases it can, and Dettleff *et al.* succeeded in creating such a liquefaction shock wave. They used three fluorocarbons, which are safer to use than hydrocarbons that could also show the same behaviour. An incident shock wave was reflected off a glass window closing the end of the tube. The reflected shock wave caused liquefaction which was observed through the window.

The formation of liquid was observed in various ways, but direct photographs through the glass proved to be the most remarkable. The photographs show numerous small ring structures in the liquid. Initially these were interpreted as spherical bubbles but closer scrutiny showed that they were toroidal bubbles, which implies they are the cores of vortex rings. The explanation seems to be that the shock must have some internal structure, perhaps related to nucleation of the liquid, that leads to 'droplet' non-uniformities. These droplets impinge on the liquid behind the shock on much the same way as the drops described in § 2.5 fall into water and cause a vortex ring motion. There are good photographs in Dettleff *et al.* (1979).

3.3. *Oscillating flow and mass transfer*

A shock propagating along a tube is a single event but it is quite easy to force a flow so that repetitive events or oscillations are set up and these can involve shock waves. The Hartmann–Sprenger tube is such a case. It is a closed tube and the oscillations are caused by a supersonic or possibly subsonic jet of gas directed at the open end. As well as very considerable noise generation, there is a rapid accumulation of heat at the closed end of the tube. For example, experiments with wooden tubes lead to charring of the wood. Brocher & Maresca (1973) describe experiments in tubes of differing materials with different gases.

Gentler oscillations, in either open or closed pipes are the basis of many musical instruments. The excitation of the various modes of oscillation occurs in various ways. One way, relevant to flutes and organs, is by eddies shed from a sharp edge. This is reviewed by Fletcher (1979) and, rather strangely, a preceding review in the same volume (Rockwell & Naudascher 1979) is on the same topic. Even more peculiar is the fact that they have only two references in common. More on musical instruments can be found in Kent (1977) and Smith & Mercer (1979).

Oscillatory flow, often in combination with a net mean flow, is characteristic of flow in many biological pipes and tubes; for example consider your windpipe and arteries. Expertise in fluid dynamics which has developed in engineering fields has been applied to many biological systems in the last twenty years. It was rather amusing to find the Director of the Royal Aircraft Establishment writing on the swimming of fish (Lighthill 1960), but it is a logical extension of aeronautical studies. Now it seems that engineering can learn from biological applications.

Such an example was brought to the attention of readers of this journal by Sobey (1980) and Stephanoff, Sobey & Bellhouse (1980). Their aim is to understand why a device developed by Bellhouse *et al.* (1973) for oxygenating blood worked so well. The device allowed oxygen to diffuse through a membrane into blood in a corrugated channel. The diffusion resistance of the device is only 10 % more than that of the membrane itself.

The blood flows in a channel, each wall of which has transverse ripples caused by the membrane taking up a curved from between each transverse support. Numerical computations and boundary layer calculations (Sobey, 1980) are supported by experiment (Stephanoff *et al.* 1980). These show that with each oscillation the flow into an expansion of the channel follows the wall at first, then separates and forms an eddy. For steady flow such an eddy would impede mass transfer, but with an appreciable oscillatory component, particularly one causing flow reversal, the changing flow sweeps the eddy to the centreline of the channel allowing a very considerable enhancement of mass transfer.

All this occurs for maximum Reynolds numbers of $O(100)$. The same type of transfer enhancement may well occur with much larger Reynolds numbers but then the greatly increased mixing of turbulent flows renders the enhancement less important.

This type of device could be valuable for the multitude of industrial processes which involve mass transfer in viscous fluids, and particularly so for the more common requirement of heat transfer in heat exchangers.

3.4. *Blowing hot, cold and luminous*

More complicated flows in pipes lead to further possibilities. If swirl occurs all sorts of phenomena appear. Batchelor's (1967) book gives an indication of the complications in §7.5. The flow cannot always accommodate a change in pipe cross-section smoothly and this or other circumstances can lead to the phenomenon of vortex breakdown which also occurs in the leading edge vortex of delta wings as Batchelor shows (figure 7.5.7). A review of vortex breakdown is given by Hall (1972) and recent experimental measurements are described by Faler & Leibovich (1978).

If the air is fed into a tube tangentially to form a strong vortex flow, and the mean flow each side of the entry is forced to differ, temperature differences arise. In particular, in a Ranque–Hilsch vortex tube one end, at some distance from the entry, has a large or circumferential orifice and the other end, near the entry has a small centrally placed orifice. The air escaping from the large orifice is heated and the air from the small opening is colder than the incoming air. Escudier, Bornstein & Zehnder (1980) give detailed flow measurements with water in a vortex tube closed at the entry end. Even so there is a return axial velocity toward the closed end in some of their measurements, and most show a marked minimum velocity, on the axis of the tube. It would be interesting to try and compare these profiles with the various exact viscous solutions of Donaldson & Sullivan (1960). An account of theory and experiment including temperature effects is given by Sibulkin (1962). Another example of disjoint work on a subject occurs here, Escudier *et al.* (1980) and Sibulkin (1962) have no references in common. The device is used for refrigeration in some circumstances but its efficiency is too low for widespread use in that area. Papers which are more practically orientated are Metenin (1960) and Parulekar (1961).

If a vortex tube is driven with air at a supersonic velocity, then Lavan & Fejar (1965) report that the centre of the tube glows. This only occurred when the test section was made of transparent and non-conducting material. No luminescence was observed in a metal tube. A study of the phenomena indicated that it is due to a glow discharge. An electric field is caused by drops of condensed water carrying charge to the tube walls. The only snag with this hypothesis seems to be that the efficiency of charge separation appears to be much higher than might be expected from theoretical estimates.

3.5. *Other warm devices*

Another ingenious tube is the 'heat pipe'. This provides efficient heat transfer for moderate distances with small temperature differences. The heat is carried by a vapour which condenses at the cooler end. The resulting liquid returns by capillary action along a wick contained within the tube. Full details are given in the monograph by Chisholm (1971).

When fuel is burnt it is usually desirable to enhance its rate of burning in some way, and at the same time to guide combustion products away from the incoming air supply. The inventor of the chimney found a very effective device to achieve both these aims. My own childhood experience suggests that the chimney may have been discovered by observation of the effects of and on a fire at the base of a hollow tree. Modern practice on dispersal of combustion products has lead to the construction of chimneys higher than 350 metres.

Similar simplicity for effecting rapid combustion and directing the combustion products is found in a less wellknown invention (E. P. Peregrine, private communication). It again consists of a pipe with no moving parts. The pipe has a wide almost spherical combustion chamber, which with an exhaust pipe of suitable length forms a Helmholtz resonator. The inlet which is opposite to the exhaust is partially obstructed and short. Combustion is an oscillatory process and a large amount of fuel can be burned in a small space. If appropriate the exhaust could also provide thrust. The main drawback of this device is that it is so noisy as to be devoid of practical value.

4. Epilogue

There is much in fluid mechanics to fascinate us. Clearly an illustrated book is needed to do the subject justice. Who can fail to admire at least some of the shapes that arise from the interaction of fluids with solids? The sweeping curves of sand dunes arise from a direct interaction. The aerodynamic shapes of seagulls and thistledown arise from an evolutionary interaction. Technological evolution has led us to some elegant ships and aircraft. Of these, yachts ought to be the most respected; they use both the sea and the air, and occupy a position which nature shuns. The Portuguese man-o'-war, *Physalia physalia*, and one of its close relatives are the only creatures I am aware of which live almost entirely on the surface of the oceans.

Finally, a fluid mechanics brainteaser. The two photographs in figure 9 can have the same, very specific, title. What is it?

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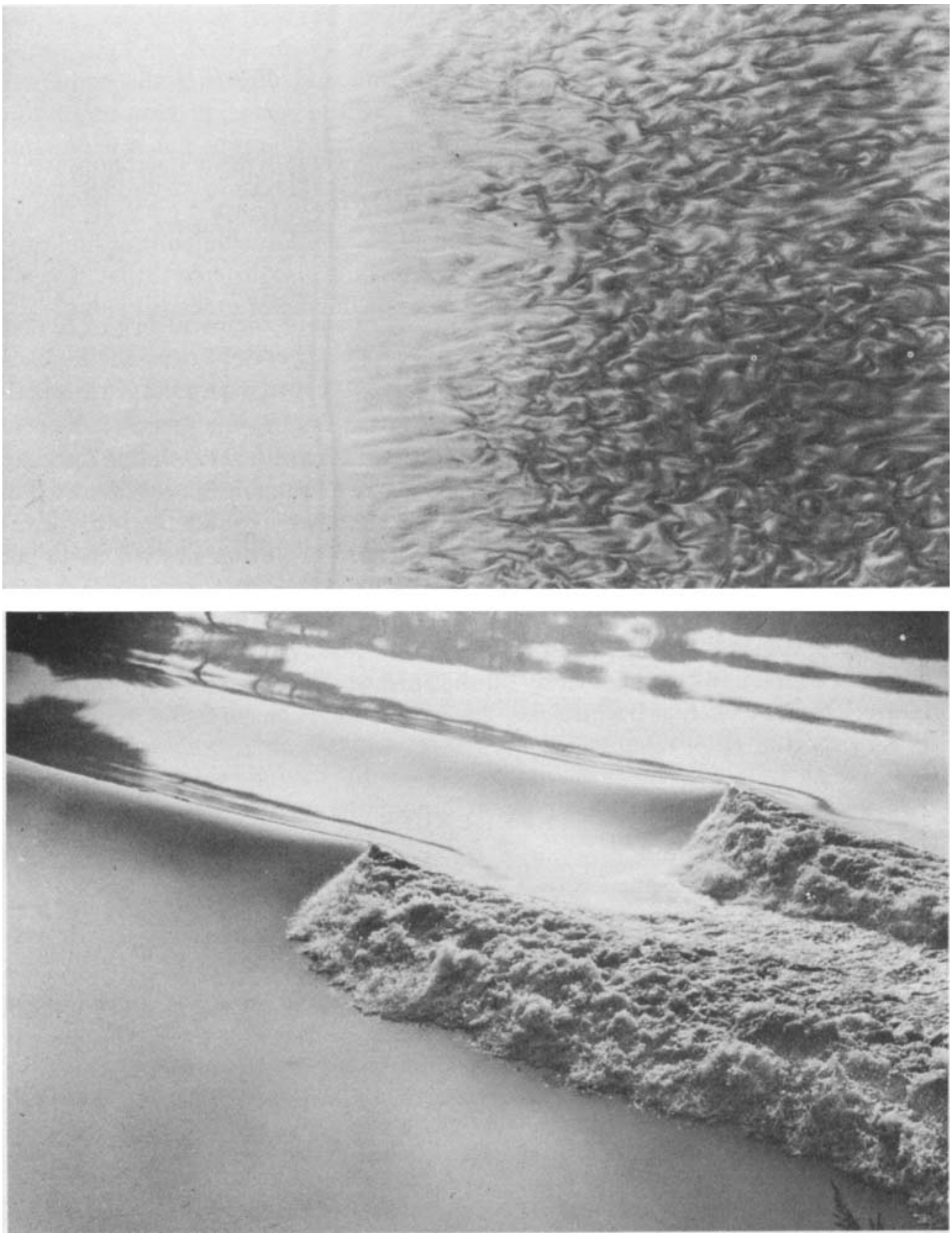

(a)

(b)

Figure 9. (a) and (b) are both 'An undular bore generating some turbulence'. (a) is a bore with capillary waves on a few millimetres depth of water. Small flake-like aluminium particles in the water help to show the flow patterns. (b) is a tidal bore on the River Severn, with gravity waves which break in the shallower water near the shore.

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